Endogenously Imperfect Risk Sharing and Financial Amplification

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Abstract

Why do financial contracts leave entrepreneurs exposed to aggregate risk even when contracts can be made fully state-contingent? I develop a financial accelerator model with a moral hazard problem in which private benefit of low effort is non-monetary and incentive compatibility holds state by state. Since the marginal value of entrepreneurial net worth is countercyclical, the incentive constraint tightens procyclically. Entrepreneurs thus take advantage of this tradeoff by offloading more idiosyncratic risk at the cost of taking on more aggregate risk. Imperfect aggregate risk sharing therefore emerges endogenously from the interaction between state-dependent incentives and risk sharing, rather than from contractual restrictions. This exposure amplifies aggregate shocks: entrepreneurial net worth becomes more sensitive to macroeconomic conditions, compressing borrowing capacity procyclically. Quantitatively, state-contingent contracting does not eliminate the financial accelerator, and the model generates substantial and persistent amplification of investment and output.

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1 Introduction

Recessions weaken firms' balance sheets, depressing cash flows, eroding asset values, and tightening access to external finance. These financial headwinds, in turn, depress investment and deepen the downturn, generating the feedback loop known as the financial accelerator, which lies at the heart of modern macro-finance (Bernanke and Gertler (1989); Carlstrom and Fuerst (1997)). Yet the strength of this mechanism depends not only on the severity of the initial shock or the tightness of financial constraints, but also on how financial contracts allocate aggregate risk. When repayments can be made contingent on publicly observable aggregate conditions, borrowers and lenders can share this risk rather than concentrate it on borrowers' balance sheets. Indeed, theoretical work shows that such risk sharing can substantially attenuate, or even eliminate, financial amplification (Krishnamurthy (2003); Carlstrom et al. (2016); DiTella (2017)).

This observation raises a puzzle. Modern financial markets offer firms a rich array of instruments and contractual arrangements to transfer and redistribute aggregate risk. Many of the aggregate conditions driving firms' cash flows and asset values are publicly observable, which makes it difficult to attribute firms' persistent large exposure to aggregate risk solely to an inability to write contingent contracts. Explaining this persistence requires understanding the incentives and frictions that shape how financial contracts allocate aggregate risk. Why, then, do financial contracts leave so much aggregate risk concentrated on borrowers' balance sheets?

This paper offers an answer that does not rely on market incompleteness, differences in risk aversion, or exogenous restrictions on the contract space. I embed hidden-effort moral hazard in a standard financial accelerator model and allow loan repayments to depend on both idiosyncratic returns and the realized aggregate return to capital. Because entrepreneurs can secretly exert low effort and obtain a private benefit, the contract must leave them sufficient agency rent to induce high effort.

The incentive-compatibility constraint must hold state by state, and the rent required to satisfy it depends on the marginal value of entrepreneurial net worth, which varies with aggregate conditions. Consequently, the terms on which entrepreneurs share idiosyncratic risk to lenders also vary with the aggregate state, influencing how much aggregate risk they retain. The optimal contract therefore need not distribute aggregate fluctuations proportionately between entrepreneurs and lenders, even when repayments can be made contingent on aggregate outcomes. Imperfect aggregate risk sharing, and the resulting financial amplification, emerge from the interaction between moral hazard and the state dependence of the marginal value of net worth, rather than

from an exogenously restricted set of financial contracts.

I formalize this mechanism in a dynamic general-equilibrium model that builds on the Bernanke et al. (1999) financial accelerator but substitutes costly state verification with hidden-effort moral hazard in the spirit of Innes (1990). The optimal contract remains risky debt, but its face value varies with the realized aggregate return to capital. Despite this state dependence, constant returns to scale imply that the entrepreneur's value function is linear in net worth, with a time-varying slope capturing the marginal value of internal funds. This homogeneity result makes the model tractable: aggregate dynamics can be characterized by entrepreneurial net worth and aggregate productivity, which allows me to quantify how incentive constraints shape financial amplification.

To quantify the role of incentive compatibility in financial amplification, I compare the main model, in which the incentive constraint must hold state by state, with a benchmark in which it is imposed only in expectation. Both environments allow repayments to be aggregate state-contingent; they differ only in how the incentive constraint restricts the contract. The quantitative results reveal markedly different dynamics. Under the static-IC benchmark, the lender's return closely comoves with aggregate conditions, thereby sharing the aggregate risk proportionately between entrepreneurs and lenders, which largely eliminates the balance sheet effect by mitigating fluctuations in leverage and spreads. Output, investment, and consumption consequently respond much like they would in a frictionless real business cycle model. Under state-contingent IC, by contrast, aggregate risk are disproportionately concentrated on entrepreneurs' balance sheets, which creates a large balance sheet effect accompanied by consistent declines in leverage and the risk premium. Investment and output responses are substantially amplified and more persistent. Thus, the way incentive compatibility is imposed can determine whether state-contingent contracting largely neutralizes or sustains the financial accelerator.

These findings caution against treating state-contingent lending as a stand-alone tool for financial stability. Whether aggregate-indexed contracts dampen financial amplification depends on whether the underlying incentive constraints allow aggregate risk to be shifted away from financially constrained borrowers. Financial regulation should therefore consider not only how contracts index repayments to aggregate conditions, but also how agency frictions determine who ultimately bears the risk.

Related Literature. This paper's results are related to a literature showing that once financial contracts can be indexed to aggregate shocks, the financial accelerator largely disappears. Krishnamurthy (2003) introduces insurance markets into a three-period,

Kiyotaki and Moore (1997) model in which borrowing is secured by collateral, and shows that state-contingent debt indexed to aggregate shocks eliminates the feedback from collateral values to investment, reducing collateral amplification to zero. Di-Tella (2017) obtains an analogous result in the continuous-time setting of Brunnermeier and Sannikov (2014): allowing contracts to condition on the aggregate state completely eliminates the financial amplification generated by productivity shocks. Carlstrom et al. (2016) show that the privately optimal contract in the Bernanke et al. (1999) environment features debt repayment indexed nearly one-for-one to the aggregate return on capital, so that default rates barely respond to innovations in capital returns; this indexation alone dramatically dampens the financial accelerator. My model nests this logic as a special case: under an expectational static incentive constraint, indexing debt repayment to the aggregate return on capital similarly eliminates the financial accelerator. This paper demonstrates, however, that this result does not generally extend to state-contingent incentives. When the agency problem requires the incentive compatibility constraint to bind in every aggregate state, state-contingent contracts can still leave substantial aggregate risk concentrated on the entrepreneur's balance sheet. Thus, the effectiveness of state-contingent contracting in eliminating financial amplification depends not only on the availability of indexation, but also on the underlying structure of the incentive constraint.

A second strand restores aggregate risk concentration by relaxing some other primitive: Bocola and Lorenzoni (2023) and Jeenas (2024) appeal to households' relatively higher risk aversion, driven in part by uninsurable idiosyncratic labor income risk, so that lenders are unwilling to absorb aggregate risk even when offered the chance to. Asriyan (2021) instead derives limited risk sharing from information and trading frictions in secondary markets for aggregate-contingent claims. Hartman-Glaser and Hébert (2020) generate limited indexation from adverse-selection concerns about the quality of an aggregate index. My mechanism is complementary to, but distinct from, all of these: I hold preferences, information, and the space of feasible contracts fixed and unrestricted, and instead show that the interaction between a hidden-effort moral hazard friction and the entrepreneur's marginal value of net worth is, by itself, able to generate imperfect aggregate risk sharing.

The remainder of the paper proceeds as follows. The rest of this section relates the paper to existing explanations for imperfect aggregate risk sharing. Section 2 lays out the model, states the optimal contracting result, and derives the comparative statics linking continuation value, debt capacity, and leverage. Section 3 calibrates the model and reports impulse responses under transitory and persistent productivity shocks, comparing the state-contingent and static IC environments, and discusses policy im-

plications. Section 4 concludes.

2 The Model

The model incorporates the microeconomics of debt contracting into an otherwise standard real business cycle framework à la Bernanke et al. (1999) (henceforth BGG) and Carlstrom et al. (2016) (henceforth CFP).

Time. Time is discrete, infinite, and indexed by t.

Agents. The economy is populated by five sectors of agents: Households, New capital producers, Lenders, Final goods producers, and Entrepreneurs.

Goods. There are two types of goods: Final goods and Capital goods. Final goods, can be consumed, stored as deposits, or used as input of the investment technology to produce capital. Capital goods can be used, combined with labor through the production technology, to produce final goods.

2.1 Households

The representative household is infinitely lived, risk-averse, discounts future at the rate β, and derives utility from consumption and disutility from supplying labor. The household maximizes discounted sum of expected utility $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$, with preference given by

$$U(C_t, L_t) = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \frac{L_t^{1+\eta}}{1+\eta} \tag{1}$$

where σ_c captures the relative risk aversion, η represents the inverse of the elasticity of labor supply, and C_t and L_t is consumption and labor supply at date t. The household budget constraint is given by

$$C_t + \text{Dep}_t \leq w_t L_t + R_t^d \, \text{Dep}_{t-1} + \Pi_t \tag{2}$$

The household decides the supply of labor at real wage w_t, and chooses the level of deposits Dep_t, which are then used by the lender to finance entrepreneurs' capital holding (more details below). As I will present later, the lender's return on the portfolio of loans is realized at time t, and this return is passed on to the households, so the gross return of time $t-1$ deposits R_t^d is realized at time t. Different from BGG, R_t^d is not a predetermined value, but is assumed to be conditional on aggregate shocks. [1] The household owns shares in the final goods producers, new capital producers and lenders. In equi-

[1] In this sense, the deposits are essentially bank equity, and the households are the sole shareholder of the lender-bank.

librium only new capital producers will generate profits Π_t that is transferred to the household.

The household's optimization conditions are given by

$$-\frac{U_{L,t}}{U_{c,t}} = w_t \tag{3}$$

$$E_t\left[M_{t+1}R^d_{t+1}\right] = 1 \tag{4}$$

where $M_{t+1} := \beta\frac{U_{c,t+1}}{U_{c,t}}$ is the household's stochastic discount factor.

2.2 New Capital Producers

New capital producers have an investment technology subject to convex adjustment cost. At date t, one inputs $I_t\vartheta(\frac{I_t}{I_{ss}})$ units of final goods to produce I_t units capital that are then sold at price Q_t. New capital producers decide the amount of new capital to be produced in order to maximize profit as the net gain of investment, i.e.,

$$\Pi_t = \max_{I_t} Q_t I_t - I_t\vartheta(\frac{I_t}{I_{ss}}) \tag{5}$$

Following BGG, I include adjustment costs to permit variations in investment and hence variations in the price of capital, which in turn lead to fluctuations in entrepreneurial net worth. I assume in steady state $\vartheta(1) = 1$, $\vartheta'(1) = 0$, $\vartheta''(1) = \vartheta_0 > 0$, so that the steady state capital price is normalized to one. In the quantitative section, the convex investment cost function is specified as $\vartheta\left(\frac{I_t}{I_{ss}}\right) = 1 + \frac{\vartheta_0}{2}\left(\frac{I_t}{I_{ss}} - 1\right)^2$.

2.3 Lenders

The representative lender, owned by and taking deposits from households, provides loans to entrepreneurs. I assume only one-period loans are available. [2] Each loan, matching the representative lender with one entrepreneur, contains both (economy-wise) aggregate risk and (entrepreneur-wise) idiosyncratic risk. Due to the Law of Large Numbers and the assumption that there is a continuum of borrowers, the loan portfolio held by the lender is fully diversified so that only aggregate risk remains and is compensated in the gross loan rate. The free entry condition ensures the lender makes zero profit, so the gross loan rate R^l_{t+1} must equal the deposit rate R^d_{t+1}.

[2] One typical explanation for my focus on one-period contract, as in BGG, would be there is enough anonymity so past credit records are not tractable, and entrepreneurs may die and exit the market.

2.4 Final Good Producers

At the start of time t, final good producers hire labor L_t and rent effective capital K_t^f in the competitive market at real wage w_t and capital rental rate r_t respectively to produce final good. They behave in a competitive way and have the same constant-return-to-scale production technology.

$$Y_t = A_t(K_t^f)^\alpha L_t^{1-\alpha} \tag{6}$$

where A_t is the total factor productivity (TFP), whose realization is publicly observed at the start of time t.

The optimization conditions are

$$mpk_t := \alpha Y_t/K_t = r_t \tag{7}$$

$$mpl_t := (1-\alpha)Y_t/L_t = w_t \tag{8}$$

where mpk and mpl denote the marginal product of capital and labor respectively.

2.5 Entrepreneurs and the Loan Contract

2.5.1 Demography and Technology

Entrepreneurs play the key role in the model. There is a continuum of risk-neutral entrepreneurs who share the same discount rate β as households. I assume that entrepreneurs are the sole accumulators of capital. They earn capital rent as a fraction of production, as well as the resale value of depreciated capital. At the start of period t, entrepreneur i has net worth $N_{i,t}$, accumulated through past production and capital holdings. To avoid self-financing in the long run, I assume that each entrepreneur faces a probability $(1-\gamma)$ of death in each period.[3] In addition, I assume that each entrepreneur's expected return to internal funds from capital holdings always exceeds the inverse of the discount rate, β^{-1}. As a result, surviving entrepreneurs always postpone consumption indefinitely, whereas those who die consume all their net worth before exiting the economy. Each exiting entrepreneur is then replaced by a new entrepreneur with a trivial amount of initial wealth to initiate economic activity. This lump-sum transfer is sufficiently small to be ignored in the dynamics of aggregate entrepreneurial wealth. Surviving entrepreneurs therefore always wish to lever up to purchase as much capital as possible.

[3] As in BGG, I assume that each entrepreneur has a finite life expectancy, facing a constant hazard rate. The finite horizon is intended to capture the ongoing entry and exit of firms, as well as to preclude self-financing, whereby entrepreneurs gradually accumulate sufficient net worth to finance their activities internally.

Suppose that entrepreneur i's capital holding stock at date t is $K_{i,t+1}$. At the prevailing capital price Q_t, net borrowing is given by $Q_t K_{i,t+1} - N_{i,t}$, and the leverage ratio is defined as $\kappa_{i,t} := \frac{Q_t K_{i,t+1}}{N_{i,t}}$. Each entrepreneur has a unique investment technology that transforms 1 unit of *raw capital* into ω units of *effective capital* ready for production, where ω is an idiosyncratic random variable (discussed in more detail below). At the start of the next period, once all risks have been realized, the entrepreneur rents out the effective capital to final goods producers and sells the depreciated capital. As in BGG, I assume that depreciated capital must be sold after production and then repurchased at the start of the next period, so that agency problems affect the whole capital stock rather than just marginal investment.

2.5.2 Financing and Frictions

To finance their capital holding, entrepreneurs use internal funding, entrepreneurial net worth, and external funding in the form of loans from lenders. External finance takes the form of a one-period contract with promised repayment at time $t+1$. The ability to repay the loan depends on end-of-period cash flow, which is determined by both final goods production and the idiosyncratic investment technology. More precisely, one unit of raw capital costs Q_t at date t. Conditional on the idiosyncratic technology $\omega_{i,t+1}$, it generates capital rent $r_{t+1}\omega_{i,t+1}$ and the resale value of the depreciated capital, $\omega_{i,t+1}Q_{t+1}(1-\delta)$, at date $t+1$, where r_{t+1} is the capital rental rate defined as in (7), and δ is the geometric depreciation rate of capital. Define the intertemporal return from holding one dollar unit of capital as $\frac{(r_{t+1}+Q_{t+1}(1-\delta))\cdot\omega_{i,t+1}}{Q_t}$. This return can be decomposed into the product of two factors: the *aggregate return* to capital R^k_{t+1} and entrepreneur i's *idiosyncratic return* $\omega_{i,t+1}$. The aggregate return to capital is given by

$$R^k_{t+1} = \frac{r_{t+1} + Q_{t+1}(1-\delta)}{Q_t} \tag{9}$$

which is driven by the TFP shock A_{t+1} and is therefore interpreted as the source of aggregate risk.

The idiosyncratic return to capital $\omega_{i,t+1}$ captures a disturbance to entrepreneur i's investment process that transforms raw capital into effective capital, and is independent of the aggregate return. Unlike the aggregate return, which is fully determined by aggregate states revealed by total productivity, the idiosyncratic return depends on the entrepreneur's individual performance. Entrepreneurs may deliberately reduce project profitability in the absence of appropriate incentives. Specifically, entrepreneurs can exert unobservable effort $e \in e_H, e_L$, where $e = e_L$ (low effort) yields a private benefit of B per unit of capital, whereas $e = e_H$ (high effort) yields no private benefit. I assume that the private benefit of low effort B is non-monetary.

Assumption 1. *The private benefit earned by entrepreneurs in exerting low effort is non-monetary.*

Effort affects the probability distribution of project returns. In particular, ω is assumed to be distributed on $[0, \infty)$, with cumulative distribution function $F_e(\omega)$ and a twice continuously differentiable density function $f_e(\omega)$. High effort improves the return distribution in the sense of first-order stochastic dominance. I further assume that the likelihood ratio $\frac{f_{e_H}(\omega)-f_{e_L}(\omega)}{f_{e_H}(\omega)}$ is strictly increasing in ω, so that $f(\cdot)$ satisfies the monotone likelihood ratio property (MLRP). Finally, low effort is not efficient: the per-unit-scale private benefit from low effort, B, is strictly lower than the expected improvement in returns, defined as $\varphi := E[\omega|e_H] - E[\omega|e_L]$. Thus, $B < \varphi$.

2.5.3 Contracting Problem

Assumption 2. *Contracts are allowed to be aggregate state-contingent, and in particular, entrepreneurs choose hidden effort on observing the realized aggregate states.*

At the start of period t, upon learning that they have survived, entrepreneurs make take-it-or-leave-it offers to the representative lender to obtain external financing in the amount $Q_t K_{i,t+1} - N_{i,t}$. Since both the aggregate return and the idiosyncratic return are observable and contractible, the repayment to the lender can be made contingent on both returns. Specifically, the repayment is given by $P(\omega_{i,t+1}, R^k_{t+1}) \cdot Q_t K_{i,t+1}$, where P can be interpreted as *pledgeable income*, that is, the fraction of the realized return on capital holdings that can be pledged to the lender. Following Innes (1990), I impose two restrictions on the repayment structure. First, Limited Liability (LL) requires that both the entrepreneur and the lender enjoy limited liability, such that $0 \leq P(\omega_{i,t+1}, R^k_{t+1}) \leq \omega_{i,t+1} R^k_{t+1}$. Second, Contractual Monotonicity (MON) requires that repayment be weakly increasing in the idiosyncratic return. Formally, $P(\omega_{i,t+1} + \epsilon, R^k_{t+1}) - P(\omega_{i,t+1}, R^k_{t+1}) \geq 0$, $\forall \epsilon > 0$.[4]

In summary, the bilateral loan contract signed at time t between entrepreneur i and the lender specifies $\{\kappa, P\}$, comprising two elements: (i) the capital purchase, expressed as the leverage ratio $\kappa_{i,t} \equiv \frac{Q_t K_{i,t+1}}{N_{i,t}}$; and (ii) the per-unit repayment scheme $P(\omega_{i,t+1}, R^k_{t+1})$, which is contingent on both the idiosyncratic and aggregate returns.

2.6 Optimal Financial Structure

At the start of date t, forward-looking entrepreneur i, with accumulated net worth $N_{i,t}$, faces an exit shock. With probability $(1 - \gamma)$, the entrepreneur exits and consumes

[4] If P is differentiable with respect to the second component, then require $\frac{\partial P}{\partial \omega} \geq 0$.

all accumulated net worth, setting $c^e_{i,t} = N_{i,t}$. With probability γ, the entrepreneur survives, uses the entire net worth to finance capital holdings, and derives utility from future consumption.

Define $V(\Gamma_{i,t}; A_t)$ as the time-t value of entrepreneur i's discounted sum of consumption, conditional on the exogenous state variable A_t and the endogenous state variables $\Gamma_{i,t}$ inherited from the past, before the realization of the exit shock. Total factor productivity, A_t, is the only exogenous state variable in the model. The vector $\Gamma_{i,t}$ collects the relevant endogenous state variables, which may generally include net worth and capital stock. However, I will show later that $\Gamma_{i,t} = \{N_{i,t}\}$, so that net worth is the only endogenous state variable relevant to an individual entrepreneur's optimization problem.

The value function V is the solution to the Bellman equation:

$$V(\Gamma_{i,t}; A_t) = (1-\gamma)c^e_{i,t} + \beta\gamma \max_{\kappa,A,P,e} E_t[V(\Gamma_{i,t+1}; A_{t+1})] \tag{10}$$

subject to lenders' participation constraint (PC) such that their expected discounted future payoff is weakly greater than the capital amount they contribute today

$$E_t[M_{t+1}P(\omega_{i,t+1}, R^k_{t+1})Q_tK_{i,t+1}; A_{t+1}] \geq Q_tK_{i,t+1} - N_{i,t} \tag{11}$$

and entrepreneur's incentive compatibility constraint (IC) such that conditional on survival, one's expected discounted future utility under high hidden effort is weakly higher than that under low effort

$$\beta\gamma V(\Gamma_{i,t+1}|e_H; A_{t+1}) \geq \beta\gamma V(\Gamma_{i,t+1}|e_L; A_{t+1}) \tag{12}$$

as well as limited liability (LL) and contractual monotonicity (MON) constraints

$$0 \leq P(\omega_{i,t+1}, R^k_{t+1}) \leq \omega_{i,t+1}R^k_{t+1}, \quad P(\omega_{i,t+1}+\epsilon, R^k_{t+1}) \geq P(\omega_{i,t+1}, R^k_{t+1}) \tag{13}$$

Note that the participation constraint holds in expectation, while the incentive constraint holds state-by-state, for entrepreneurs are allowed to make hidden effort choice conditional on observing the realized aggregate states and hence incentives are to be provided in any state.

Conditional on the aggregate return on capital R^k_{t+1} and idiosyncratic risk $\omega_{i,t+1}$, entrepreneur i's payoff from holding capital at the start of date $t+1$ is

$$\left[\omega_{i,t+1}R^k_{t+1} - P(\omega_{i,t+1}, R^k_{t+1})\right]\kappa_{i,t}N_{i,t} \tag{14}$$

Similarly, the state-by-state lender return is

$$R^l_{i,t+1} = P(\omega_{i,t+1}, R^k_{t+1}) \frac{\kappa_{i,t}}{\kappa_{i,t} - 1} \tag{15}$$

Under the assumption that payoffs are linear in the project's outcome and both contracting parties are effectively risk-neutral, Innes (1990) demonstrates that the optimal contract between entrepreneur and lender is risky debt. Now I extend this result to general equilibrium with aggregate risk.

Result 1. The optimal financial structure takes the form of aggregate state-contingent risky debt, and the entrepreneur's value function is linear in net worth. All entrepreneurs choose the same loan terms ex ante, with loan size scaled proportionally to their respective net worth. Specifically, the value function and optimal repayment scheme satisfy

$$V_{i,t} = J_t \cdot N_{i,t},$$

where J_t denotes the marginal value of net worth, and

$$P(\omega_{i,t+1}, R^k_{t+1}) = \min \left\{ \omega_{i,t+1}, D_{t+1}(R^k_{t+1}) \right\} R^k_{t+1},$$

where $D_{t+1}(R^k_{t+1})$ is the state-contingent debt face value.

The detailed proof is provided in Appendix A.1 and follows a guess-and-verify approach. It proceeds in two steps. First, I conjecture that the optimal contract takes the form of aggregate state-contingent debt and show that, under this conjecture, the entrepreneur's value function is linear in net worth. Second, I verify that debt is indeed optimal given the linearity of the entrepreneur's value function in net worth.

In the main text, I focus on the first part of the entrepreneur's constrained optimization problem and derive the associated optimality conditions, taking the optimality of debt as given. Since the optimal contract takes the form of risky debt, its repayment structure can be summarized by a state-contingent face value, denoted by $D_{i,t+1}$. This face value is defined with respect to the realization of the idiosyncratic return and may depend on the aggregate return on capital, R^k_{t+1}.

To characterize this contract, define pledgeable income as the expected share of the project outcome accruing to lenders:

$$\mathcal{L}_e(D) = (1 - \chi) \int_0^D \omega dF_e(\omega) + D(1 - F_e(D)) \tag{16}$$

where χ denotes the cost of default, formally defined as the deadweight loss in the

repayment transfer process when the lender's repayment falls below the contractual debt face value. [5]

Similarly, the entrepreneur's expected monetary return is given by

$$\mathcal{E}_e(D) = \int_D^\infty (\omega - D) dF_e(\omega) \tag{17}$$

Since high effort is always incentive-compatible, the entrepreneur's expected payoff—and hence next period's net worth—is given by

$$N_{i,t+1} = R^k_{t+1} \mathcal{E}_{e_H}(D_{i,t+1}) \kappa_{i,t} N_{i,t} \tag{18}$$

Finally, the lender's rate of return is given by

$$R^l_{i,t+1} = \mathcal{L}_{e_H}(D_{i,t+1}) R^k_{t+1} \frac{\kappa_{i,t}}{\kappa_{i,t} - 1} \tag{19}$$

In the remainder of this subsection, I use a guess-and-verify approach to establish the linearity of the entrepreneur's value function. I begin by conjecturing that

$$\begin{aligned} V_{i,t} &= J_t \cdot N_{i,t} \\ &= J_t \cdot N_{i,t} + B \cdot \mathcal{I}(e_{i,t} = e_L) R^k_t \kappa_{i,t-1} N_{i,t-1} \\ &= J_t \cdot \tilde{N}_{i,t} \end{aligned} \tag{20}$$

where

$$\tilde{N}_{i,t} = N_{i,t} + \frac{B \cdot \mathcal{I}(e_{i,t} = e_L)}{J_t} R^k_t \kappa_{i,t-1} N_{i,t-1} \tag{21}$$

defines entrepreneur i's net worth adjusted for non-monetary private benefits at date t, and J_t denotes the marginal utility of net worth. This formulation distinguishes between monetary wealth and non-monetary private benefits, allowing the entrepreneurial value function $V_{i,t}$ to be decomposed into two components. The first component, $J_t N_{i,t}$, represents the marginal utility of wealth multiplied by the monetary net worth accumulated from the previous capital holding, as described in (18). The second component, $B \cdot \mathcal{I}(e_{i,t} = e_L) R^k_t \kappa_{i,t-1} N_{i,t-1}$, captures the off-equilibrium non-monetary benefit of exerting low hidden effort. This component is zero in equilibrium, since entrepreneurial incentives are sufficiently strong to ensure that $e_{i,t} = e_H$. Unlike monetary wealth, non-monetary private benefits cannot be accumulated as wealth; instead, they enter the value function directly and are not scaled by the marginal utility of wealth. Thus, an additional unit of non-monetary private benefit contributes directly to the value func-

[5] The cost of default can be microfounded as an auditing cost in models that derive optimal contracts from a costly state verification problem (see BGG). This assumption is also widely used without an explicit microfoundation to capture the well-documented fact that default is costly.

tion, independently of variations in the marginal utility of wealth.

Under this conjecture, the entrepreneur's constrained optimization problem can be reformulated. The original Bellman equation (10) is given by

$$J_t \cdot N_{i,t} = (1-\gamma)N_{i,t} + \beta\gamma \max_{\kappa_{i,t}, D_{i,t+1}, e} E_t \left[J_{t+1} \cdot N_{i,t+1} + B \cdot \mathcal{I}(e_{i,t+1} = e_L) R^k_{t+1} \kappa_t N_{i,t}; A_{t+1} \right]$$

Substituting the conjectured value function in (20) into the Bellman equation yields

$$J_t = (1-\gamma) + \beta\gamma \max_{\kappa_{i,t}, D_{i,t+1}, e} \kappa_{i,t} E_t \left[(J_{t+1}\mathcal{E}_e(D_{i,t+1}) + B \cdot \mathcal{I}(e_{i,t+1} = e_L)) R^k_{t+1}; A_{t+1} \right] \tag{22}$$

The lender's participation constraint (PC) becomes

$$E_t \left[M_{t+1} R^k_{t+1} \mathcal{L}_{e_H}(D_{i,t+1}) \kappa_{i,t}; A_{t+1} \right] \geq \kappa_{i,t} - 1 \tag{23}$$

while the entrepreneur's incentive compatibility constraint (IC) becomes

$$\beta\gamma\kappa_{i,t} \left[J_{t+1}\mathcal{E}_{e_H}(D_{i,t+1}) \right] R^k_{t+1} \geq \beta\gamma\kappa_{i,t} \left[J_{t+1}\mathcal{E}_{e_L}(D_{i,t+1}) + B \right] R^k_{t+1}, \qquad \forall A_{t+1}. \tag{24}$$

Under the maintained assumptions, the entrepreneur's contracting problem is concave, so its solution is characterized by the Lagrangian first-order conditions. Let $\Lambda_{i,t}$ denote the Lagrange multiplier on the lender's participation constraint (PC), and let $\mu_{i,t+1}$ denote the state-contingent multiplier on the entrepreneur's incentive compatibility constraint (IC). Because the risky debt contract specified above automatically satisfies limited liability and contractual monotonicity, the relevant first-order conditions and binding constraints are:

$$\begin{aligned} 0 = {} & \beta\gamma\kappa_{i,t} J_{t+1} \mathcal{E}'_{e_H}(D_{i,t+1}) + \Lambda_{i,t} M_{t+1} \kappa_{i,t} \mathcal{L}'_{e_H}(D_{i,t+1}) \\ & + \beta\gamma\kappa_{i,t}\mu_{i,t+1} J_{t+1} \left[\mathcal{E}'_{e_H}(D_{i,t+1}) - \mathcal{E}'_{e_L}(D_{i,t+1}) \right]. \end{aligned} \tag{25}$$

$$\beta\gamma E_t R^k_{t+1} [J_{t+1}\mathcal{E}_{e_H}(D_{i,t+1})] + \Lambda_{i,t} [E_t M_{t+1} R^k_{t+1} \mathcal{L}_{e_H}(D_{i,t+1}) - 1] = 0 \tag{26}$$

$$E_t \left[M_{t+1} R^k_{t+1} \mathcal{L}_{e_H}(D_{i,t+1}) \kappa_{i,t} \right] = \kappa_{i,t} - 1 \tag{27}$$

$$J_{t+1} \left[\mathcal{E}_{e_H}(D_{i,t+1}) - \mathcal{E}_{e_L}(D_{i,t+1}) \right] = B, \qquad \forall A_{t+1}. \tag{28}$$

Equations (25) and (26) characterize the optimal choices of $D_{i,t+1}$ and $\kappa_{i,t}$, respectively. Since $D_{i,t+1}$ depends on variables realized at time $t+1$, equation (25) holds state-by-state. Equations (27) and (28) impose the binding lender participation constraint and entrepreneur incentive compatibility constraint, respectively. The former

holds in expectation, whereas the latter holds state-by-state.

Comparing equations (25) and (26) implies that $D_{i,t+1} = D_{t+1}$, so the debt face value is constant across entrepreneurs. Similarly, equation (27) implies that $\kappa_{i,t} = \kappa_t$, so the leverage ratio is also identical across entrepreneurs. The recursive structure of the problem further implies that $D_{i,t} = D_t$ and $\Lambda_{i,t} = \Lambda_t$. Returning to the Bellman equation (22), the fact that the optimal choices of $\kappa_{i,t}$ and $D_{i,t+1}$ are common across entrepreneurs implies that J_t is also common across entrepreneurs. This verifies the conjecture that the value function takes the form $V_{i,t} = J_t \cdot N_{i,t}$, where the slope coefficient J_t varies over time but is constant across entrepreneurs at any given time.

Using (26) and (27) in (22), the Bellman equation thus becomes

$$J_t = (1 - \gamma) + \Lambda_t \tag{29}$$

and using (29) in (22) we get the dynamics of Λ_t

$$\Lambda_t = \beta\gamma\kappa_t E_t[1 - \gamma + \Lambda_{t+1}]R^k_{t+1}\mathcal{E}_{e_H}(D_{t+1}) \tag{30}$$

The contracting problem derived in the above section, together with the constant-returns-to-scale technology, implies that capital demand is proportional to entrepreneurial net worth. Thus, all entrepreneurs receive the same loan package, up to the scale of their individual net worth, so that differences across entrepreneurs affect their investment level but not the terms of their financing. As a result, the distribution of net worth across entrepreneurs need not be tracked separately. The dynamics of the economy can therefore be fully characterized by a single endogenous state variable, aggregate entrepreneurial net worth N_t, and a single exogenous state variable, total factor productivity A_t. Aggregating across entrepreneurs yields the law of motion for aggregate entrepreneurial net worth:

$$N_{t+1} = \gamma R^k_{t+1}\mathcal{E}_{e_H}(D_{t+1})\kappa_t N_t \tag{31}$$

2.7 Comparative Statics

The contracting problem yields a simple relationship between the entrepreneur's marginal value of net worth and external financing capacity. I first characterize the direct incentive channel, holding other aggregate variables and prices fixed, and then discuss its implications for the business cycle.

The incentive channel. From the binding incentive compatibility constraint (28),

$$J_{t+1}\left[\mathcal{E}_{e_H}(D_{t+1}) - \mathcal{E}_{e_L}(D_{t+1})\right] = B.$$

The term in brackets measures the incremental residual payoff available to the entrepreneur under high effort rather than low effort. The incentive constraint therefore requires the entrepreneur to retain sufficient future rents to offset the private benefit B of shirking. Equivalently,

$$\Delta_{\mathcal{E}}(D_{t+1}) := \mathcal{E}_{e_H}(D_{t+1}) - \mathcal{E}_{e_L}(D_{t+1}) = \frac{B}{J_{t+1}}.$$

A higher marginal value of net worth J_{t+1} reduces the payoff differential required to induce high effort. Under the maintained MLRP and regularity conditions, $\Delta_{\mathcal{E}}(D)$ is decreasing in D. Hence, a higher J_{t+1} allows the lender to claim a larger debt face value while preserving the entrepreneur's incentives.

A larger debt face value increases pledgeable income, thereby relaxing the external financing constraint. Through the binding lender participation constraint (27), this expands the amount of capital that can be financed relative to entrepreneurial net worth. The direct incentive channel can thus be summarized as

$$J_{t+1} \uparrow \implies D_{t+1} \uparrow \implies \mathcal{L}_{e_H}(D_{t+1}) \uparrow \implies \kappa_t \uparrow .$$

These relationships isolate the effect of the marginal value of net worth through the binding incentive compatibility constraint. In general equilibrium, the resulting contractual choices also depend on the stochastic discount factor, the return on capital, and other aggregate variables.

Cyclical implications and risk sharing. The comparative statics above have direct implications for the cyclical allocation of aggregate risk. Equation (29) expresses the marginal value of net worth as

$$J_t = (1 - \gamma) + \Lambda_t,$$

where Λ_t is the shadow cost of external finance. Since Λ_t is countercyclical, so is J_t. Applying the incentive channel established above, a higher marginal value of net worth in downturns permits a larger debt face value and expands external financing capacity relative to net worth. Conversely, in booms, a lower J_t requires a larger payoff differential to sustain high effort, constraining the debt face value and pledgeable income. Within this channel, the debt face value and the leverage ratio κ_t therefore move countercyclically.

Equivalently, the required payoff differential $\frac{B}{J_t}$ is procyclical: it rises in booms and falls in downturns. The incentive constraint consequently links the cyclical value of

entrepreneurial net worth to the terms on which entrepreneurs can borrow. This relationship matters for aggregate dynamics because changes in financing capacity affect how realized returns translate into entrepreneurial net worth and, in turn, future investment.

As such, the key economic insight is that the availability of state-contingent contracts does not guarantee frictionless risk sharing. Although the debt face value can adjust to aggregate conditions, its state dependence is constrained by incentive compatibility: repayments cannot be chosen solely to redistribute aggregate risk, because the entrepreneur must retain sufficient residual payoffs to induce high effort. The resulting allocation of risk therefore depends on the interaction between the incentive constraint and the cyclical marginal value of net worth. When these contractual responses are incorporated into the aggregate law of motion for entrepreneurial net worth, they determine how aggregate shocks translate into changes in net worth and subsequent financing capacity. Imperfect risk sharing, and the associated financial amplification, thus arise endogenously from incentive constraints rather than from an exogenously imposed restriction on the state contingency of financial contracts.

2.8 An Alternative Benchmark: Static Incentive Compatibility

To isolate the role played by *state-by-state* incentive provision, I construct an alternative contracting environment identical to the baseline model except for the timing of the entrepreneur's effort choice.

Assumption 2′. *The entrepreneur commits to a single effort level at the time the contract is signed, before the aggregate state A_{t+1} is realized, rather than choosing effort after observing A_{t+1} as in Assumption 2.*

Because effort is chosen once, incentive compatibility becomes a single aggregate condition rather than a continuum of state-by-state conditions: the entrepreneur's *expected* discounted payoff from committing to high effort must weakly exceed the expected payoff from committing to low effort,

$$\beta\gamma\, E_t\Big[\kappa_t\, J_{t+1}\, \mathcal{E}_{e_H}(D_{t+1})\, R^k_{t+1}\Big] \;\geq\; \beta\gamma\, E_t\Big[\kappa_t\big(J_{t+1}\, \mathcal{E}_{e_L}(D_{t+1}) + B\big)\, R^k_{t+1}\Big]. \tag{32}$$

Since B remains a per-unit-scale, non-monetary benefit (Assumption 1), it scales with R^k_{t+1} exactly as the monetary residual claim does, so R^k_{t+1} *cannot* be cancelled here as it was in the state-by-state case (24), since cancellation there relied on the comparison holding for each realization individually. Rearranging (32) yields the static incentive

constraint

$$E_t\Big\{J_{t+1}\big[\mathcal{E}_{e_H}(D_{t+1})-\mathcal{E}_{e_L}(D_{t+1})\big]\,R^k_{t+1}\Big\}\;\geq\;B\,E_t\big[R^k_{t+1}\big].$$

Equation (32) replaces the continuum of constraints (24) with one constraint: incentive compatibility need only hold *on average* across realizations of A_{t+1}, not in every realization.

Contracting problem. The entrepreneur's problem is otherwise unchanged: choose κ_t and a repayment schedule $D_{t+1}(\cdot)$ to maximize expected continuation value subject to the participation constraint (23) and the aggregate incentive constraint (32). Let Λ_t denote the multiplier on (23), as before, and let μ_t denote the multiplier on (32). The value function remains linear in net worth, $V_{i,t}=J_tN_{i,t}$, by the same guess-and-verify argument as in Section 2.6. Crucially, μ_t is a *single, time-t scalar*, common across realizations of A_{t+1}, as in contrast to the baseline model, where (24) holding state-by-state generates a state-contingent multiplier $\mu_{i,t+1}(A_{t+1})$.

The first-order condition for D_{t+1} in a given realization of A_{t+1} is

$$0=\beta\gamma\kappa_t\,J_{t+1}\,\mathcal{E}'_{e_H}(D_{t+1})+\Lambda_tM_{t+1}\kappa_t\,\mathcal{L}'_{e_H}(D_{t+1})+\mu_t\,J_{t+1}\big[\mathcal{E}'_{e_H}(D_{t+1})-\mathcal{E}'_{e_L}(D_{t+1})\big],\qquad\forall A_{t+1},\tag{33}$$

together with (23) and (32) holding with equality.

Comparison with the baseline model. Equation (33) is structurally identical to the baseline condition (25), with one essential difference: μ_t is constant across states, whereas $\mu_{i,t+1}$ in (25) varies with A_{t+1} and is pinned state-by-state through the binding constraint (28). In the baseline model, a tightening of the incentive problem in a particular state feeds *directly* into that state's debt capacity, since (28) determines $D_{t+1}(A)$ entirely from $J_{t+1}(A)$. Under Assumption 2′, incentive provision is instead priced once, ex ante, at a single shadow cost μ_t; any remaining state-dependence of $D_{t+1}(A)$ is inherited solely from the state-contingent prices $J_{t+1}(A)$, $M_{t+1}(A)$, and $R^k_{t+1}(A)$ entering (33) and (23), the same channel through which the private optimal contract of Carlstrom et al. (2016) indexes repayment to aggregate conditions.

2.9 Competitive Equilibrium

The competitive equilibrium is described by the variables $\{C_t, C^e_t, L_t, I_t, Q_t, K_{t+1}, R^l_t, R^d_t, \kappa_t, D_{t+1}, \Lambda_t\}$ such that

(i) deposit market clears

$$R^d_t=R^l_t\tag{34}$$

(ii) labor market clears

$$-\frac{U_{L,t}}{U_{c,t}} = mpl_t \tag{35}$$

(iii) capital price and consumption allocation are such that contracting terms optimize entrepreneur's problem

$$E_t\left[M_{t+1}R^k_{t+1}\mathcal{L}_{e_H}(D_{t+1})\frac{\kappa_t}{\kappa_t - 1}\right] = 1 \tag{36}$$

$$(1-\gamma+\Lambda_{t+1})\left[\mathcal{E}_{e_H}(D_{t+1}) - \mathcal{E}_{e_L}(D_{t+1})\right] = B, \tag{37}$$

$$\Lambda_t = \beta\gamma\kappa_t E_t[1-\gamma+\Lambda_{t+1}]R^k_{t+1}\mathcal{E}_{e_H}(D_{t+1}) \tag{38}$$

$$R^l_{t+1} = R^k_{t+1}\mathcal{L}_{e_H}(D_{t+1})\frac{\kappa_t}{\kappa_t - 1} \tag{39}$$

(iv) capital goods clear

$$K_{t+1} = \Upsilon(D_t)K_t(1-\delta) + I_t \tag{40}$$

$$Q_t = \vartheta(\frac{I_t}{I_{ss}}) + (\frac{I_t}{I_{ss}})\vartheta'(\frac{I_t}{I_{ss}}) \tag{41}$$

(v) resources and final goods clear

$$Q_t K_{t+1} = \gamma[Q_t(1-\delta) + mpk_t]\mathcal{E}_{e_H}(D_t)K_t\kappa_t \tag{42}$$

$$C^e_t = (1-\gamma)[Q_t(1-\delta) + mpk_t]\mathcal{E}_{e_H}(D_t)K_t \tag{43}$$

$$C_t + C^e_t + I_t\vartheta(\frac{I_t}{I_{ss}}) = A_t(\Upsilon(D_t)K_t)^\alpha L_t^{1-\alpha} \tag{44}$$

where I have used $M_{t+1} := \beta\frac{U_{c,t+1}}{U_{c,t}}$, $\kappa_t := \frac{Q_t K_{t+1}}{N_t}$, $R^k_{t+1} := \frac{r_{t+1}+Q_{t+1}(1-\delta)}{Q_t}$, $\Upsilon(D) := \mathcal{E}(D) + \mathcal{L}(D)$, $mpl_t := (1-\alpha)Y_t/L_t$, $mpk_t := \alpha Y_t/(\Upsilon(D_t)K_t)$, $Y_t := A_t(\Upsilon(D_t)K_t)^\alpha(L_t)^{1-\alpha}$, and definitions of $\mathcal{L}_e(D)$ and $\mathcal{E}_e(D)$ as in equation (16)-(17).

3 Quantitative Analysis

In this section, I first specify the exogenous aggregate shocks in this economy, and then calibrate the model to match some key aggregate moments. Finally I investigate the model's quantitative implications.

3.1 Functional Forms and Aggregate Shocks

Following Carlstrom et al. (2016), the idiosyncratic risk follows a log-normal distribution: $\log(\omega) \sim \mathcal{N}(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2)$, where σ_ω is the standard deviation of $\log(\omega)$ and thus the expectation of ω under high effort is normalized to 1, $E_H[\omega] = 1$.

I assume the natural logarithm of aggregate productivity, $\ln A_t$ follows the process

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0, s_A^2) \tag{45}$$

where ρ_A measures the persistence of total productivity shock, $\epsilon_{A,t}$ and $\epsilon_{\sigma,t}$ denotes exogenous shock to total productivity, and s_A is the standard deviation of the shock.

3.2 Calibration

I calibrate the model at quarterly frequency, and the calibrated parameters are summarized in Table 1. The first part of parameters is predefined and borrowed from BGG and Carlstrom et al. (2016). The households' utility is set to be logarithmic in consumption, i.e, $\sigma_c = 1$, and the elasticity of labor is set to be 3, i.e., $\eta = 1/3$. The capital share of the Cobb-Douglas production is $\alpha = 0.35$.

The parameters in the second part are determined by matching the first moments of macroeconomic quantities and prices. I set the time discount rate $\beta = 0.99$ to generate a steady state annual growth rate of 1 percent. The quarterly capital depreciation rate is $\delta = 0.025$ such that the annual depreciation rate is 10 percent. The entrepreneurs' survival probability γ is set to 0.94 such that the average life expectancy is 10 years for Compustat firms. The investment adjustment costs $\vartheta_0 = 0.5$ such that in the steady state the price of capital is equal to unity.

As for the credit-related parameters, I jointly calibrate them to match four empirical moments: (i) a steady-state quarterly spread between Z and R^d of 0.5 percent, (ii) a quarterly default rate of 0.75 percent, (iii) a recovery rate of 35 percent following debt default, and (iv) a leverage ratio of $\kappa = 2$. Given a private benefit of $B = 0.3$, these targets imply a survival rate of $\gamma = 0.93$, a standard deviation of the idiosyncratic shock of $\sigma_\omega = 0.28$, a default cost of $\chi = 0.625$, and a high-effort improvement in expected productivity of $\varphi = 0.35$.

Figure 1 presents impulse response functions to a 1 percent i.i.d. TFP shock, with $\rho_A = 0$. The responses reveal a sharp contrast between the two contracting environments. Under the static IC, the shock has little persistence: leverage and the risk premium remain essentially unchanged, as the lender's return adjusts one-for-one to contemporaneous conditions. The shock raises output, consumption, and investment while modestly reducing leverage, but these effects quickly dissipate, consistent with the pri-

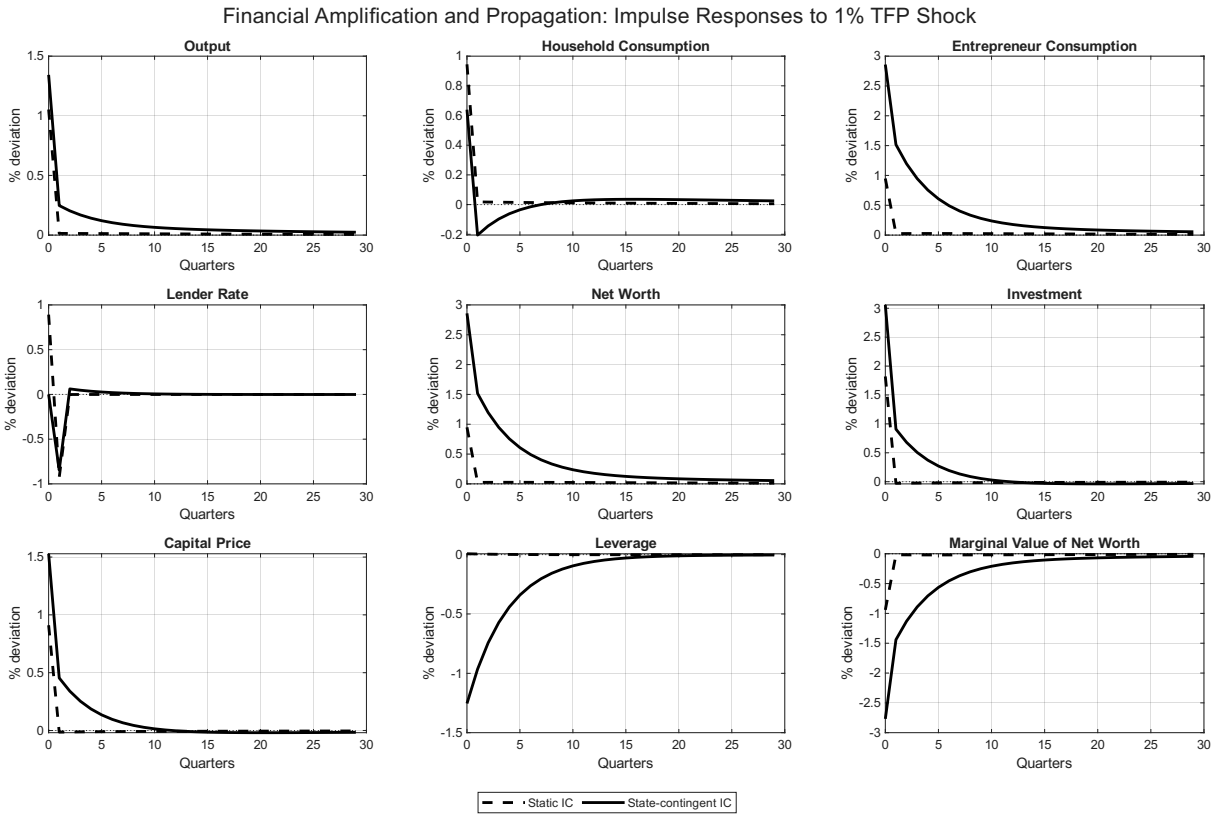


Figure 1: Independently and Identically Distributed TFP Shock

vately optimal contracting regime as Carlstrom et al. (2016). Under the state-contingent IC, by contrast, the shock generates a sharp increase in entrepreneurial net worth and a corresponding persistent decline in leverage. The resulting improvement in the entrepreneur's balance sheet amplifies the responses of output and investment, with the effects decaying only gradually as net worth returns toward its steady state. Thus, despite the absence of persistence in the underlying TFP shock, the state-contingent IC generates persistent real effects through the dynamics of entrepreneurial net worth.

Figure 2 considers an autocorrelated TFP shock. Under the static IC, lender repayment adjusts contemporaneously with the shock, leaving leverage and the risk premium essentially unchanged and limiting the role of financial amplification. By contrast, under the state-contingent IC, the persistent TFP shock generates a sustained increase in entrepreneurial net worth, shifting consumption toward the entrepreneur and gradually reducing leverage and the risk premium. The resulting improvement in financing conditions amplifies the responses of investment and output, with these effects persisting over the 30-quarter horizon as net worth adjusts gradually toward its steady state. Thus, while both contracting environments transmit the persistence of the TFP shock to real activity, only the state-contingent IC generates an additional financial amplification channel through endogenous movements in net worth and leverage.

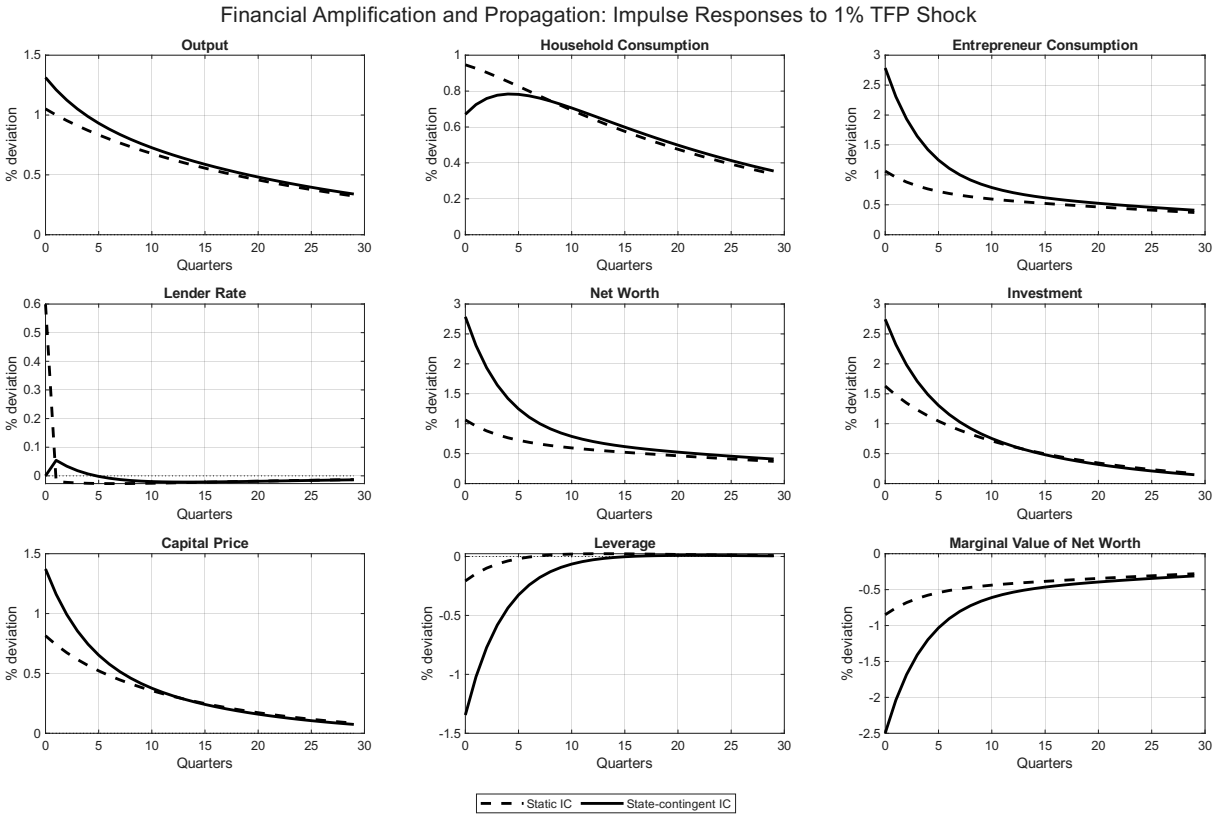


Figure 2: Persistent TFP Shock

3.3 Policy Remarks

The quantitative results highlight that allowing state-contingent repayment is not, by itself, sufficient to eliminate balance-sheet amplification. What matters is how financial contracts allocate aggregate risk between borrowers and lenders once incentive compatibility is taken into account. In the present environment, the binding incentive constraint links the terms of the debt contract to the marginal value of entrepreneurial net worth, which varies with aggregate conditions. This creates a trade-off in risk allocation. State-contingent contracting can allow entrepreneurs to transfer more idiosyncratic risk to lenders, but at the cost of leaving entrepreneurs more exposed to aggregate fluctuations through reduced insurance of their net worth. Thus, better insurance against idiosyncratic risk need not translate into better insurance against aggregate risk. The resulting balance-sheet channel depends not on state contingency per se, but on how contractual flexibility interacts with incentive constraints to determine the allocation of aggregate risk.

More broadly, this mechanism underscores the importance of the microfoundations for understanding the macroeconomic effects of financial contracting. Different contracting environments may generate similar borrowing constraints and external financ-

ing premia in steady state, yet imply markedly different responses to aggregate shocks. In the present model, the marginal value of net worth is linked to the shadow cost of external finance, whose cyclical variation changes the incentive rents required to implement high effort. These changes, in turn, affect the debt face value, pledgeable income, and leverage, shaping how aggregate shocks translate into changes in entrepreneurial net worth and subsequent financing capacity. Consequently, the macroeconomic implications of contract indexation or state-contingent repayment cannot be inferred from the borrowing constraint alone. They depend on the underlying source of the agency friction and, crucially, on how it governs the trade-off between idiosyncratic risk and aggregate risk sharing.

This mechanism has implications for both macroprudential regulation and the design of state-contingent debt. Under Basel III, the countercyclical capital buffer is designed to strengthen the banking system against risks accumulating during periods of excessive credit growth and to help preserve the supply of credit during downturns (Basel Committee on Banking Supervision (2010)). Separately, the IMF has examined GDP-linked and other state-contingent sovereign debt instruments as a means of aligning debt service with repayment capacity and preserving policy space following adverse shocks (International Monetary Fund (2017)). Although the present model does not directly evaluate either policy, it highlights an important consideration for their design: making financial claims contingent on aggregate conditions does not guarantee that aggregate risk is effectively transferred away from financially constrained borrowers. For policymakers, contractual risk-sharing arrangements and macroprudential safeguards should therefore be viewed as potentially complementary tools. The effectiveness of state-contingent debt depends on the incentives governing risk allocation, while macroprudential assessments must account for the balance-sheet vulnerabilities that can persist even when contracts are flexible.

4 Conclusion

Financial contracts serve not only to intermediate capital between borrowers and lenders, but also to allocate risk. This paper examines whether state-contingent repayment can eliminate financial amplification when entrepreneurs face hidden-effort moral hazard. The analysis shows that contractual flexibility is not sufficient. Even when repayments can depend on both idiosyncratic returns and the aggregate return to capital, incentive compatibility constrains how aggregate risk can be allocated. Because the rents required to sustain high effort depend on the marginal value of entrepreneurial net worth, which varies with aggregate conditions, the optimal contract need not fully in-

sure entrepreneurs against aggregate fluctuations. Imperfect aggregate risk sharing can therefore emerge endogenously, rather than from an exogenously restricted set of financial contracts.

Quantitatively, the model highlights the macroeconomic importance of how incentive compatibility is imposed. When the incentive constraint must hold only in expectation, state-contingent repayment largely neutralizes fluctuations in leverage and credit spreads, and output, investment, and consumption behave much like they do in a frictionless real business cycle model. When the constraint must hold state by state, the same productivity shock generates a sharper and more persistent response in entrepreneurial net worth, more persistent declines in leverage and the risk premium, and more amplified and persistent responses of investment and output. These differences arise even though both environments allow repayments to depend on aggregate conditions. They reflect how the incentive constraint limits the use of contractual flexibility to redistribute risk.

The results underscore that the macroeconomic consequences of financial contracting cannot be inferred from the degree of state contingency alone. They also depend on the underlying agency friction and on how it shapes the allocation of aggregate risk. More broadly, financial models with similar steady-state financing constraints may generate substantially different business-cycle dynamics because their incentive constraints govern risk sharing in different ways. Understanding financial amplification therefore requires attention not only to the terms of financial contracts, but also to the frictions and incentives that determine who ultimately bears aggregate risk.

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A Proofs

A.1 Proof of Result 1.

I present a formal proof of Result 1. To simplify the analysis further, I assume in this part that surviving entrepreneurs derive utility only from collecting the next-period payoff from their capital holdings. Dropping all other terms, including J_{t+1}, the entrepreneur's objective function becomes

$$\max_{\kappa,P,e} N_{t+1}^i$$

subject to constraints from equation (11) to (13).

I suppress the time and idiosyncratic risk indices throughout the remainder of the proof. The goal is to show that the optimal contract takes the form of risky debt, with the repayment to the lender given by

$$P(\omega, R^k) = \min \omega, D(R^k) \cdot R^k,$$

where ω is the realized idiosyncratic return and D is the promised non-default return, or face value, of the performance-sensitive debt. Since D depends on a variable realized at time $t+1$, the debt face value is contingent on the observed aggregate state R^k.

The proof proceeds by showing that, for any feasible repayment schedule satisfying incentive compatibility, limited liability, and contractual monotonicity, one can construct a debt contract of the form above that satisfies the same constraints and strictly dominates the original contract in terms of incentives: the entrepreneur's incentive constraint is easier to satisfy under the debt contract. That is, for any $P(\omega, R^k)$ there exhits a risky debt contract $\min\{\omega, D(R^k)\}$ such that

$$\int_0^\infty P(\omega, R^k) f(\omega|e_H) d\omega = \int_0^\infty \min\{\omega, D(R^k)\} \cdot R^k f(\omega|e_H) d\omega, \quad \forall R^k$$

So the debt repayment must meet lender's participation constraint

$$\begin{aligned} 1 &= E[M(R^k) \int_0^\infty P(\omega, R^k) f(\omega) d\omega | \phi(R^k)] \frac{\kappa}{\kappa - 1} \\ &= E[M(R^k) \int_0^\infty \min\{\omega, D\} \cdot R^k f(\omega) d\omega | \phi(R^k)] \frac{\kappa}{\kappa - 1} \end{aligned}$$

and also equalize entrepreneur's payoff

$$\int_0^\infty \int_0^\infty (\omega R^k - P(\omega, R^k)) f(\omega|e_H) d\omega \phi(R^k) dR^k =$$
$$\int_0^\infty \int_0^\infty (\omega R^k - \min\{\omega, D\} \cdot R^k) f(\omega|e_H) d\omega \phi(R^k) dR^k$$

where $M(R^k)$ is the lender's stochastic discount factor, fully determined by aggregate state R^k, and $f(\cdot)$, $g(\cdot)$ and $\phi(\cdot)$ are the probability density function of random variable ω and R^k respectively.

Consider the non-trivial case in which

$$P(\omega, R^k) \neq \min\{\omega, D\} \cdot R^k$$

on a set with strictly positive measure. I show that any repayment $P(\omega, R^k)$ is strictly dominated by the corresponding debt repayment $\min\{\omega, D\} \cdot R^k$ in the sense of providing incentives. That is to say, entrepreneur's incentive to shirk is higher with repayment $P(\omega, R^k)$, i.e.,

$$\int_0^\infty \int_0^\infty (\omega R^k - P(\omega, R^k)) f(\omega|e_L) d\omega \phi(R^k) dR^k >$$
$$\int_0^\infty \int_0^\infty (\omega R^k - \min\{\omega, D\} \cdot R^k) f(\omega|e_L) d\omega \phi(R^k) dR^k$$

To establish this result, I prove the following stronger sufficient condition:

$$\int_0^\infty (\omega R^k - P(\omega, R^k)) f(\omega|e_L) d\omega > \int_0^\infty (\omega R^k - \min\{\omega, D\} \cdot R^k) f(\omega|e_L) d\omega, \quad \forall R^k$$

Consider the function $h(\omega, R^k) \equiv (\omega R^k - P(\omega, R^k)) - \max(\omega - D, 0) \cdot R^k$, and I know

$$0 = \int_0^\infty h(\omega, R^k) f(\omega|e_H, a) d\omega, \quad \forall R^k$$

Given aggregate state R^k, $h(\cdot)$ as a function of ω has mean zero, so it must be strictly positive and strictly negative in sets of states with positive measure. Moreover, because of limited liability, $\omega R^k - P(\omega, R^k) \geq 0$, and of contractual monotonicity $\frac{\partial P}{\partial \omega} \geq 0$, and hence $\frac{dh}{d\omega} = R^k - \frac{\partial P}{\partial \omega} - R^k I(\omega \geq D)$ must be negative for some large ω. As a consequence, there must exist $\omega_1(R^k) \in (0, +\infty)$ such that $h(\omega) \geq 0$ when $\omega \leq \omega_1(R^k)$ and $h(\omega) < 0$

when $\omega > \omega_1(R^k)$. This way,

$$
\begin{aligned}
0 &= \int_0^\infty h(\omega) f(\omega|e_H) d\omega \\
&= \int_0^\infty h(\omega) \frac{f(\omega|e_H)}{f(\omega|e_L)} f(\omega|e_L) d\omega \\
&= \int_0^{\omega_1(R^k)} h(\omega) \frac{f(\omega|e_H)}{f(\omega|e_L)} f(\omega|e_L) d\omega + \int_{\omega_1(R^k)}^\infty h(\omega) \frac{f(\omega|e_H)}{f(\omega|e_L)} f(\omega|e_L) d\omega) g(s|e_H) d\omega \\
&< \int_0^{\omega_1(R^k)} h(\omega) \frac{f(\omega_1|e_H)}{f(\omega_1|e_L)} f(\omega|e_L) d\omega + \int_{\omega_1(R^k)}^\infty h(\omega) \frac{f(\omega_1|e_H)}{f(\omega_1|e_L)} f(\omega|e_L) d\omega \\
&= \frac{f(\omega_1(R^k)|e_H)}{f(\omega_1(R^k)|e_L)} (\int_0^{\omega_1(R^k)} h(\omega) f(\omega|e_L) d\omega + \int_{\omega_1(R^k)}^\infty h(\omega) f(\omega|e_L) d\omega) \\
&= \frac{f(\omega_1(R^k)|e_H)}{f(\omega_1(R^k)|e_L)} \int_0^\infty h(\omega) f(\omega|e_L) d\omega
\end{aligned}
$$

By the definition of h, this establishes that the entrepreneur's expected payoff under low effort is strictly greater with the original repayment scheme than with the corresponding debt contract. Equivalently, the debt contract provides stronger incentives for high effort.

Therefore, the optimal contract, conditional on the interim signal, takes the form of risky debt.

B Model Characterization

To better understand the driving forces of the model, I employ a first-order approximation approach. Because there is only total productivity shock, first-order approximation is enough to capture the first-order effects.

B.1 Steady States

In the steady-state, equations (36) to (40) and (31) become

$$\beta\gamma R^k_{ss}[(1-\gamma+\Lambda_{ss})\mathcal{E}_{e_H}(D_{ss})] + \Lambda_{ss}[\beta R^k_{ss}\mathcal{L}_{e_H}(D_{ss}) - 1] = 0 \tag{46}$$

$$(1-\beta R^k_{ss}\mathcal{L}_{e_H}(D_{ss}))\kappa_{ss} = 1 \tag{47}$$

$$(1-\gamma+\Lambda_{ss})\mathcal{E}_{e_H}(D_{ss}) = (1-\gamma+\Lambda_{ss})\mathcal{E}_{e_L}(D_{ss}) + B \tag{48}$$

$$\gamma R^k_{ss}\mathcal{E}_{e_H}(D_{ss})\kappa_{ss} = 1 \tag{49}$$

We can solve the steady-state values of financial variables as implicit functions of the underlying distribution parameters

$$R^k_{ss}(\gamma \mathcal{E}_{e_H}(D_{ss}) + \beta \mathcal{L}_{e_H}(D_{ss})) = 1 \tag{50}$$

$$\kappa_{ss} - 1 = \frac{\beta \mathcal{L}_{e_H}(D_{ss})}{\gamma \mathcal{E}_{e_H}(D_{ss})} \tag{51}$$

$$\Lambda_{ss} = \frac{\beta(1-\gamma)}{1-\beta} \tag{52}$$

$$\mathcal{E}_{e_H}(D_{ss}) = \mathcal{E}_{e_L}(D_{ss}) + B\frac{1-\beta}{1-\gamma} \tag{53}$$

B.2 First-order Approximation

Using these steady state expressions, here I log linearize equations

$$\tilde{\kappa}_t + E_t[r^k_{t+1} + \Theta_{\mathcal{E}} \tilde{D}_{t+1}] = \lambda_t - \beta E_t \lambda_{t+1} \tag{54}$$

$$E_t[m_{t+1} + r^k_{t+1} + \Theta_{\mathcal{L}} \tilde{D}_{t+1}] = \frac{1}{\kappa_{ss} - 1}\tilde{\kappa}_t \tag{55}$$

$$\beta \lambda_t + \Theta_{\Delta\mathcal{E}} \widetilde{D}_t = 0, \tag{56}$$

where $\Theta_{\mathcal{E}} = \frac{D_{ss}\mathcal{E}'_{e_H}(D_{ss})}{\mathcal{E}_{e_H}(D_{ss})} < 0$, $\Theta_{\mathcal{L}} = \frac{D_{ss}\mathcal{L}'_{e_H}(D_{ss})}{\mathcal{L}_{e_H}(D_{ss})} > 0$, and $\Theta_{\Delta\mathcal{E}} \equiv \frac{D_{ss}\left[\mathcal{E}'_{e_H}(D_{ss}) - \mathcal{E}'_{e_L}(D_{ss})\right]}{\mathcal{E}_{e_H}(D_{ss}) - \mathcal{E}_{e_L}(D_{ss})} < 0$.

The binding incentive constraint implies

$$\beta \lambda_t + \Theta_{\Delta\mathcal{E}} \widetilde{D}_t = 0. \tag{57}$$

Therefore,

$$\widetilde{D}_t = -\frac{\beta}{\Theta_{\Delta\mathcal{E}}}\lambda_t. \tag{58}$$

Substituting this expression into the linearized participation constraint yields

$$\widetilde{\kappa}_t = (\kappa_{ss} - 1) E_t \left[m_{t+1} + r^k_{t+1} - \frac{\beta \Theta_{\mathcal{L}}}{\Theta_{\Delta\mathcal{E}}} \lambda_{t+1} \right]. \tag{59}$$

Finally, substituting the expressions for $\widetilde{D}_{t+1}$ and $\widetilde{\kappa}_t$ into the linearized Bellman equation gives

$$\lambda_t = (\kappa_{ss} - 1) E_t m_{t+1} + \kappa_{ss} E_t r^k_{t+1} + \Psi_\lambda E_t \lambda_{t+1}, \tag{60}$$

where

$$\Psi_\lambda \equiv \beta \left[1 - \frac{(\kappa_{ss} - 1)\Theta_{\mathcal{L}} + \Theta_{\mathcal{E}}}{\Theta_{\Delta\mathcal{E}}} \right]. \tag{61}$$

Thus, the contracting block can be summarized by

$$\widetilde{D}_t = -\frac{\beta}{\Theta_{\Delta\mathcal{E}}}\lambda_t, \tag{62}$$

$$\widetilde{\kappa}_t = (\kappa_{ss} - 1)E_t\left[m_{t+1} + r^k_{t+1} - \frac{\beta\Theta_{\mathcal{L}}}{\Theta_{\Delta\mathcal{E}}}\lambda_{t+1}\right], \tag{63}$$

$$\lambda_t = (\kappa_{ss} - 1)E_t m_{t+1} + \kappa_{ss}E_t r^k_{t+1} + \Psi_\lambda E_t\lambda_{t+1}. \tag{64}$$